Relaxation properties of weakly coupled stochastic Ginzburg-Landau models under intense noise

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We investigate time relaxation properties of some correlation functions of stochastic Ginzburg-Landau models with weak coupling and under intense noise. Using a Feynman-Kac representation and a "high-temperature"-type approach, we study the low-lying spectrum of the generator of the dynamics, which determines the relaxation properties. We give the one-particle mass and energy-momentum dispersion curve, and also the two-particle bound-state mass, and show that both masses increase with the noise strength, in contrast with the behavior in the small noise regime.

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I. INTRODUCTION

The relevance of noise effects on dynamical systems is well known: random variables are present everywhere, improving the description of mesoscopic and macroscopic systems in physics (biology, chemistry, etc.) [1], and generating remarkable phenomena such as noise-induced phase transition [2] and stochastic resonance [3]. The significance of such effects makes the study of the basic dynamical properties of extensively used stochastic models a subject of general interest.

In this paper, we study relaxation properties of the Langevin dynamics of weakly coupled stochastic Ginzburg-Landau (GL) models in the intense noise regime, and we describe an approach inspired on high-temperature expansions. These time-dependent GL models are extensively used in physics, and frequently appear in the study of dynamical critical phenomena: e.g., for a statistical-mechanical system they may describe the time evolution of the order parameter. Here, we analyze in detail the time decay (relaxation to equilibrium) of some correlation functions studying the low-lying spectrum of the generator of the Langevin dynamics: specifically, we describe the one-particle dispersion curve and show conditions for the existence of a two-particle bound state in such a regime. The present paper is related to investigations on the role played by changes in the noise strength in the basic dynamical properties of general stochastic models. Here, we present results which contrast with the behavior in the small noise region presented in previous papers [4,5] (contrast which turns the intermediate noise regime a region of particular interest): now, under intense noise, the one-particle and the two-particle bound-state masses increase with the noise strength (we also show that the conditions for the existence of a bound state also change with the noise regime).

We emphasize that all these results are directly related to experimentally observable effects: e.g., for a magnetic system, the one-particle mass gives the time decay rate of the magnetization fluctuation, and the two-particle bound-state mass gives the relaxation rate of the fluctuations in the susceptibility (more details ahead).

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The rest of the paper is organized as follows. In Sec. II, we describe the model, the present and previous results. In Sec. III, we analyze the correlation functions working on the "high-temperature" approach. Section IV is devoted to the final comments.

II. MODEL AND RESULTS

We consider the time-dependent GL model, i.e., a system described by a scalar field $\varphi(t,\vec{x})\in\mathbb{R}$ in a lattice space $\vec{x}\in\mathbb{Z}^d$, $t\in[0,\infty)$ (we make t discrete later), with stochastic dynamics given by the Langevin equation

$$\frac{\partial}{\partial t}\varphi(\vec{x},t)=-\frac{1}{2}\nabla S(\varphi(\vec{x},t))+\eta(\vec{x},t), \tag{1}$$

where $\nabla S=\delta S/\delta\varphi$ with

$$S(\varphi)=\sum_{\vec{x}\in\mathbb{Z}^d}\left\{\frac{1}{2}\varphi(\vec{x})((-\Delta+m^2)\varphi)(\vec{x})+\lambda\mathcal{P}(\varphi(\vec{x}))\right\}, \tag{2}$$

Δ is the lattice Laplacian, λ and m are positive parameters ($\lambda\ll 1\ll m$), and $\mathcal{P}$ is an even polynomial, bounded from below. η is a Gaussian white-noise random variable with the expectations

$$E(\eta(\vec{x},t))=0,\quad E(\eta(\vec{x},t)\,\eta(\vec{y},t'))=\gamma\delta_{\vec{x},\vec{y}}\delta(t-t'),$$

γ is positive (the noise strength). We want to know the time behavior of functions $f(\varphi)$ with time evolution given by $f_t(\psi)=E(f(\varphi(t)))$, where $\varphi(0)=\psi$ is some initial condition in Eq. (1). We will skip some technical details (presented, e.g., in [6,7] and references therein). Stochastic calculus gives us

$$f_t(\psi)=e^{-tH}f(\psi),$$

$$Hf=\left\{\sum_{\vec{x}\in\mathbb{Z}^d}-\frac{1}{2}\gamma\frac{\partial^2}{\partial\varphi(\vec{x})^2}+\frac{1}{2}\frac{\partial S}{\partial\varphi(\vec{x})}\frac{\partial}{\partial\varphi(\vec{x})}\right\}f. \tag{3}$$

The generator of the dynamics H is Hermitian, positive in $L^2(d\mu)$, $d\mu\equiv e^{-S(\varphi)/\gamma}d\varphi$/normalization. The ground state is $f=1$, with zero eigenvalue. We can write H in terms of a Schrödinger-type operator

$$L=UHU^{-1}$$
$$=\sum_{\vec{x}\in\mathbb{Z}^d}\left\{-\frac{1}{2}\gamma\frac{\partial^2}{\partial\varphi(\vec{x})^2}+\frac{1}{4}\left[\frac{1}{2\gamma}\left(\frac{\partial S}{\partial\varphi(\vec{x})}\right)^2-\frac{\partial^2 S}{\partial\varphi(\vec{x})^2}\right]\right\}, \tag{4}$$

where U is the unitary operator $Uf(\varphi)=Z^{-1/2}\times\exp[-S/2\gamma]f(\varphi)$ from $L^2(d\mu(\varphi))$ to $L^2(d\varphi)$ (Z is the normalization). The Schrödinger representation suggests to us to establish a quantum-field-theory formulation for the initial time evolution problem. Hence, a Feynman-Kac representation follows [6]

$$(\Omega,\varphi(\vec{x}_1)e^{-(t_2-t_1)H}\varphi(\vec{x}_2)\cdots e^{-(t_n-t_{n-1})H}\varphi(\vec{x}_n)\Omega)_{L^2(d\mu)}$$
$$=(U\Omega,\varphi(\vec{x}_1)e^{-(t_2-t_1)L}\varphi(\vec{x}_2)\cdots e^{-(t_n-t_{n-1})L}\times\varphi(\vec{x}_n)U\Omega)_{L^2(d\varphi)}$$
$$=(\Omega,\varphi e^{-(t_2-t_1)H+i\vec{P}\cdot(\vec{x}_2-\vec{x}_1)}\times\varphi\cdots e^{-(t_n-t_{n-1})H+i\vec{P}\cdot(\vec{x}_n-\vec{x}_{n-1})}\varphi\Omega)_{L^2(d\mu)}$$
$$=\int\varphi(t_1,\vec{x}_1)\cdots\varphi(t_n,\vec{x}_n)d\rho, \tag{5}$$

where $\Omega(\varphi)=1$ is the ground state of H, $t_1\leqslant t_2\leqslant\cdots\leqslant t_n$, $t\in\mathbb{R}$, $\varphi(\vec{x})$ is the zero time field at $\vec{x}$, $\varphi=\varphi(\vec{0})$, $\vec{P}$ are momentum operators commuting with H (the infinite volume theory is translational invariant), and $d\rho=e^{-W}d\nu/\int e^{-W}d\nu$ with

$$W(\varphi)=\int_{-\infty}^{\infty}dt\sum_{\vec{x}\in\mathbb{Z}^d}\left\{\frac{\lambda}{4\gamma}\mathcal{P}'(\varphi(\vec{x},t))[(-\Delta+m^2)\varphi](\vec{x},t)+\frac{\lambda^2}{8\gamma}\mathcal{P}'(\varphi(\vec{x},t))^2-\frac{\lambda}{4}\mathcal{P}''(\varphi(\vec{x},t))\right\}, \tag{6}$$

the notation $'$ means the derivative with respect to φ, and $d\nu$ is a Gaussian measure with mean zero and covariance given by

$$\gamma C(\vec{x},t;\vec{y},t')\equiv\frac{\gamma}{(2\pi)^{d+1}}\int_{-\infty}^{\infty}dp_0\times\int_{T^d}d^dp\frac{e^{ip_0(t-t')+i\vec{p}\cdot(\vec{x}-\vec{y})}}{p_0^2+\left(\sum_{i=1}^{d}(1-\cos p_i)+\frac{m^2}{2}\right)^2}, \tag{7}$$

T^d is the torus $(-\pi,\pi]^d$.

The Feynman-Kac formula and the spectral theorem for Hermitian operators give us the connection between the spectrum of the generator of the dynamics and the behavior of the correlation functions. For the truncated two-point function (which gives the fluctuation in the magnetization for a magnetic system) $S_2(x,y)\equiv\langle\varphi(x)\varphi(y)\rangle-\langle\varphi(x)\rangle\langle\varphi(y)\rangle\equiv\int\varphi(x)\varphi(y)d\rho-\int\varphi(x)d\rho\int\varphi(y)d\rho$, $x\equiv(x_0,\vec{x})$, $x_0\equiv t\in\mathbb{R}$ (after the Feynman-Kac representation), $\vec{x}\in\mathbb{Z}^d$, direct calculations from Eq. (5) lead to

$$\tilde{S}_2(p)=\int_0^\infty\int_{T^d}\frac{2E}{E^2+(p_0)^2}\times(2\pi)^d\delta(\vec{q}-\vec{p})d(\Omega,\varphi\mathcal{E}(E,\vec{q})\varphi\Omega),$$

where $\tilde{S}_2$ is the Fourier transform of S_2, $\mathcal{E}(E,\vec{p})$ is the spectral projection associated with the operators $(H,\vec{P})$, the integral over E runs from 0 to ∞ and that over $\vec{q}$ runs in T^d. Hence, a singularity in $\tilde{S}_2$ for imaginary $p_0=ik_0$ gives a point in the spectrum (one-particle sector) of the generator of the dynamics (by the Paley-Wiener theorem, the singularity in $\tilde{S}_2$ is related to the decay of S_2). For the partially truncated four-point function (describing fluctuations in the susceptiblility for a magnetic system)

$$D(x_1,x_2;x_3,x_4)\equiv\langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle-\langle\varphi(x_1)\varphi(x_2)\rangle\langle\varphi(x_3)\varphi(x_4)\rangle$$

we have the formula

$$(\tilde{f},\tilde{D}\tilde{f})(k)=\int d^{d+1}pd^{d+1}q\overline{\tilde{f}}(\vec{p})\tilde{D}(p,q,k)\tilde{f}(\vec{q}) \tag{8}$$
$$=\int_0^\infty\int_{T^d}\frac{2E}{(k^0)^2+E^2}(2\pi)^{3d+2}\times\delta(\vec{q}-\vec{k})d(\theta(f),\mathcal{E}(E,\vec{q})\theta(f)),$$

where $\theta(\vec{\eta})=\varphi(\vec{0})\varphi(\vec{\eta})\Omega-(\Omega,\varphi(\vec{0})\varphi(\vec{\eta})\Omega)\Omega$; $\theta(f)=\sum_{\vec{x}\in\mathbb{Z}^d}f(\vec{x})\theta(-\vec{x})$; f: $\mathbb{Z}^d\to\mathbb{C}$ is an arbitrary compact function; p,q,k are the Fourier conjugate variables of $\xi=x_2-x_1$, $\eta=x_4-x_3$, and $\tau=x_3-x_2$ (due to translation invariance, D depends only on difference variables). We take above $\xi_0=\eta_0=0$. As in the case of the two-point formula, the singularities in k_0 above give information about the spectrum of H on the even subspace of states with momentum $\vec{k}$.

In [4,5], considering an approximation up to second order in λ for the computation of the one-particle mass and first order in λ for the Bethe-Salpeter (BS) kernel in the two-particle bound-state mass, the low-lying spectrum is investigated for the case of noise intensity not very large ($\gamma\leqslant m^2$). It is shown, in this region, the existence of a bound state for $d=1$ and 2 and a negative quartic term in the GL potential, and that its mass is more sensitive to changes in the noise strength than the mass of the one-particle state,

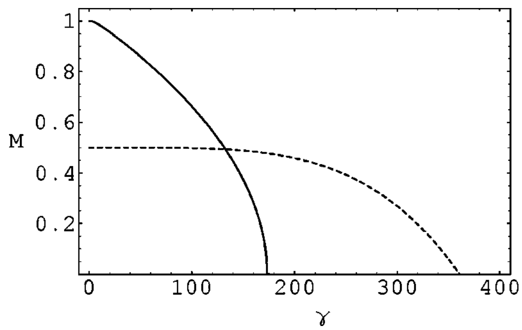

FIG. 1. Behavior in the small noise regime: curve M (one-particle mass for the dashed line, and two-particle bound-state mass for the full line) versus γ (noise intensity). The unit in the graphic is $m^2/2$, where $m^2/2$ is the one-particle "bare mass." We take above $d=1$, $m=10$, $\lambda=0.001$, $a_4=-1$, $a_6=0.1$, and $\mathcal{P}(\varphi)=(a_4/4)!:\varphi^4:+(a_6/6)!:\varphi^6:$.

decaying rapidly as we increase the noise. See Fig. 1. For suitable values of the parameters λ, m, it is predicted the existence of a mass crossover phenomenon: the two-particle bound state may become smaller than the one-particle mass for some noise intensity. It is also shown, for $d=1$ and in the ladder approximation (leading order in λ), the absence of resonances and the existence of an antibound state close to the two-particle threshold for a positive quartic term in the GL potential. The existence of this antibound state is interesting as an indication of the possibility of a bound state due to small changes in the system. We have to emphasize that in [7] we prove, for $\gamma=1$, that the spectral results obtained in the ladder approximation, up to minor changes, are maintained in the complete treatment. In [4], other phenomena are predicted, however, for large noise intensity, in a range which makes the question of the used approximation. Hence, in such a scenario, a natural and interesting question is the behavior of the system under intense noise.

In the present paper, we turn to this large noise regime: $\gamma \gg m^2$. We keep the quantum-field (Feynman-Kac) description of the problem, but now as applied for a "magnetic system" at high temperature.

Here, considering an expansion in powers of $\beta=1/\gamma$, we obtain the one-particle energy-momentum dispersion curve and establish conditions for the existence of a two-particle bound state. Taking the BS equation in the ladder approximation (leading term in the β expansion), we show that we have a bound state for large noise if the field correlations satisfy the inequality $\langle\varphi^4\rangle > 3\langle\varphi^2\rangle^2$, where

$$\langle\varphi^n\rangle = \int_{-\infty}^{\infty} \varphi^n \exp\left\{-\frac{\beta\lambda^2}{8}[\mathcal{P}'(\varphi)]^2 + \frac{\lambda}{4}\mathcal{P}''(\varphi)\right\} d\varphi/(\text{normalization}),$$

$\mathcal{P}$ is the polynomial in the GL potential, $\mathcal{P}'$ its derivative, etc. In particular, for small λ (and large γ) and $\mathcal{P}(\varphi)=(a_6/6)\varphi^6+(a_4/4)\varphi^4$, we shall have a bound state for a_4 positive or negative (a_6 positive *a priori*), and any space dimension d. The calculations for the one-particle mass give $M\simeq -\ln\beta$, and for the two-particle bound-state mass they give $M^*\simeq 2M+\ln(1-\zeta)$, where $\zeta=[\langle\varphi^4\rangle-3\langle\varphi^2\rangle^2]/[\langle\varphi^4\rangle-\langle\varphi^2\rangle^2]$, which show that both increase with the noise strength (as $\log\gamma$), in opposition to the behavior in the small noise regime.

III. HIGH-TEMPERATURE APPROACH

For the analysis in the intense noise regime we take $t\in\mathbb{Z}$ (as said, we are only interested in the low-lying spectrum), and write the previous Feynman-Kac formula as, e.g., for the two-point function,

$$S_2(u,v)=\frac{\int \varphi(u)\varphi(v)\exp\left\{-\beta\left[\lambda\tilde{W}(\varphi)+\frac{1}{2}(\varphi,C^{-1}\varphi)\right]\right\}d\nu(\phi)}{\int \exp\left\{-\beta\left[\lambda\tilde{W}(\varphi)+\frac{1}{2}(\varphi,C^{-1}\varphi)\right]\right\}d\nu(\phi)}, \tag{9}$$

where $\beta=1/\gamma$,

$$\tilde{W}(\varphi)=\frac{1}{4}\sum_{x,y}\mathcal{P}'(\varphi(x))(-\Delta+m^2)(x,y)\varphi(y), \tag{10}$$

$$(\varphi,C^{-1}\varphi)=\sum_{x,y}\varphi(x)C^{-1}(x,y)\varphi(y).$$

C is given by formula (7), and the measure $d\nu(\varphi)$ is given by products of the single spin distribution

$$d\nu(\varphi)=\prod_{x\in\mathbb{Z}^{d+1}}\exp\left\{-\frac{\beta\lambda^2}{8}[\mathcal{P}'(\varphi(x))]^2 + \frac{\lambda}{4}\mathcal{P}''(\varphi(x))\right\}d\varphi(x)/(\text{normalization}). \tag{11}$$

The expression above is formal, but we may start with the system on a finite lattice and show later the existence of the thermodynamic limit. Now our problem is described as a continuous spin system in $\mathbb{Z}^{d+1}$, with nonlocal interactions and a particular single spin distribution. The formulas connecting the correlation functions and the spectrum of the generator of the dynamics are slightly changed

$$\tilde{S}_2(p)=\int_0^\infty\int_{T^d}\frac{\sinh E}{\cosh E-\cos p_0}\times(2\pi)^d\delta(\vec{q}-\vec{p})d(\Omega,\hat{\varphi}\mathcal{E}(E,\vec{q})\hat{\varphi}\Omega)\equiv\int\frac{\sinh E}{\cosh E-\cos p_0}d\eta(E,\vec{p}),\tag{12}$$

$$(\tilde{f},\tilde{D}\tilde{f})(k)=\int d^{d+1}pd^{d+1}q\bar{\tilde{f}}(\vec{p})\tilde{D}(p,q,k)\tilde{f}(\vec{q}),\tag{13}$$

$$=\int_0^\infty\int_{T^d}\frac{\sinh E}{\cosh E-\cos k_0}\times(2\pi)^{3d+2}\delta(\vec{q}-\vec{k})d(\theta(f),\mathcal{E}(E,\vec{q})\theta(f)).$$

We turn to the analysis of $\tilde{S}_2(p)$. To determine the singularities we search for the zeroes of $\tilde{\Gamma}(p)$, which is the Fourier transform of $\Gamma(x,y)$, the correlation inverse of the two-point function $S_2(x,y)$. For simplicity, we take the polynomial in the GL potential as $\mathcal{P}(\varphi)=(a_6/6)\varphi^6+(a_4/4)\varphi^4$, but similar results follow for a more general expression $\mathcal{P}(\varphi)=\Sigma_{n=1}^N(a_{2n}/2n)\varphi^{2n}$. The zeroes of $\tilde{\Gamma}(p_0=iM(\vec{p}),\vec{p})$ give the energy-momentum dispersion curve. Simple calculations give us the result which we summarize below, for transparency.

One-particle dispersion curve: For the GL stochastic model [considering $\mathcal{P}(\varphi)=(a_6/6)\varphi^6+(a_4/4)\varphi^4$, for simplicity], in the intense noise regime $\gamma\equiv\beta^{-1}\gg m^2$, we have an isolated one-particle dispersion curve given by (up to first order in β)

$$\cosh M(\vec{p})=1+\frac{1}{4\beta\langle\varphi^2\rangle^2}+\frac{1}{4\langle\varphi^2\rangle^2}\times\left[\frac{\lambda c_1}{4}+c_2+c_3\left(\sum_{j=1}^d(1-\cos p_j)+\frac{m^2}{2}\right)\right]+\frac{1}{2}\left(\sum_{j=1}^d(1-\cos p_j)+\frac{m^2}{2}\right)^2,\tag{14}$$

where $\langle\cdot\rangle$ is the expectation with respect to the single spin distribution (11), and

$$c_1=\{a_6[\langle\varphi^8\rangle-2\langle\varphi^6\rangle\langle\varphi^2\rangle]+a_4[\langle\varphi^6\rangle-2\langle\varphi^4\rangle\langle\varphi^2\rangle]\}(2d+m^2),$$

$$c_2=\{\langle\varphi^4\rangle-2\langle\varphi^2\rangle^2\}C^{-1}(0),$$

$$c_3=a_6\langle\varphi^6\rangle\langle\varphi^2\rangle+a_4\langle\varphi^4\rangle\langle\varphi^2\rangle.$$

Hence, the one-particle mass M is given by $M(\vec{0})\simeq-\ln\beta$, which increases with the noise strength ($\gamma=1/\beta$). In fact, now the mass is generated by the noise intensity, i.e., for the spin system picture, the gap in the spectrum is due to the high temperature. In the small noise regime, the one-particle mass is close to $m^2/2$ [5], and decays as we increase the noise.

Now we analyze the partially truncated four-point function to determine the mass spectrum in the interval $(M,2M)$. Again, for clearness, we summarize the results below.

Two-particle bound state: For the considered GL stochastic model, in the intense noise regime $\gamma\gg m^2$, if the polynomial $\mathcal{P}$ in the GL potential is such that $\langle\varphi^4\rangle>3\langle\varphi^2\rangle^2$, where $\langle\cdot\rangle$ is the expectation with respect to the single spin distribution (11), then there is an isolated two-particle bound state with mass $M^*=2M+\log(1-\zeta)+O(1/\gamma)$, where M is the one-particle mass and $\zeta=[\langle\varphi^4\rangle-3\langle\varphi^2\rangle^2]/[\langle\varphi^4\rangle-\langle\varphi^2\rangle^2]$.

Sketch of proof. We use the BS Eq. [6] which writes D as $D=D_0+DKD_0$, i.e.,

$$D(x_1x_2;x_3x_4)=D_0(x_1x_2;x_3x_4)+\sum_{y_1,y_2,y_3,y_4}D(x_1x_2;y_1y_2)\times K(y_1y_2;y_3y_4)D_0(y_3y_4;x_3x_4),$$

where

$$D_0(x_1,x_2;x_3,x_4)\equiv\langle\varphi(x_1)\varphi(x_3)\rangle\langle\varphi(x_2)\varphi(x_4)\rangle+\langle\varphi(x_1)\varphi(x_4)\rangle\langle\varphi(x_2)\varphi(x_3)\rangle.$$

Again, using the relative coordinates ξ,η, and τ and the Fourier transform, the BS equation becomes

$$\tilde{D}(p,q,k)=\tilde{D}_0(p,q,k)+\frac{1}{(2\pi)^{2(d+1)}}\int_{T^{d+1}}d^{d+1}p'd^{d+1}q'\times\tilde{D}(p,p',k)\tilde{K}(p',q',k)\tilde{D}_0(q',q,k),$$

or using the notation $(\tilde{D}(k)f)(p)\equiv\int_{T^{d+1}}d^{d+1}q\times\tilde{D}(p,q,k)f(q)$,

$$\tilde{D}(k)=\tilde{D}_0(k)+\frac{1}{(2\pi)^{2(d+1)}}\tilde{D}(k)\tilde{K}(k)\tilde{D}_0(k)=\tilde{D}_0(k)[1-(2\pi)^{-2(d+1)}\tilde{K}(k)\tilde{D}_0(k)]^{-1}.$$

We restrict the analysis to the mass spectrum, i.e., $k=(k_0,\vec{k}=\vec{0})$, and take $f(p)$ depending only on $\vec{p}$. From the definition of D_0, we get

$$\tilde{D}_0(p,q,k)=(2\pi)^{d+1}[\tilde{S}_2(p)\tilde{S}_2(q)\delta(k-p-q)+\tilde{S}_2(p)\tilde{S}_2(k-p)\delta(q-p)].\tag{15}$$

Hence, $(\tilde{D}_0(k_0)f)(p)=(2\pi)^{d+1}\tilde{S}_2(p)\tilde{S}_2(k-p)[f(\vec{p})+f(-\vec{p})]$. For $f(\vec{p})=f(-\vec{p})$, we obtain

$$(\tilde{f},\tilde{D}(k_0)\tilde{f})=\int \tilde{f}(\vec{p})\Big\{2(2\pi)^{d+1}\int dp_0\tilde{S}_2(p)\times\tilde{S}_2(k_0-p_0,-\vec{p})\Big\}\left(\left[1-\frac{1}{(2\pi)^{2(d+1)}}\times\tilde{K}(k_0)\tilde{D}_0(k_0)\right]^{-1}\tilde{f}\right)(\vec{p})d^dp. \quad (16)$$

As $\{\ldots\}(k_0,\vec{p})$ above is analytic in $|\mathrm{Im}\ \ k_0|<2M$, the singularities in such a region come from where the inverse of $1-(2\pi)^{-2(d+1)}\tilde{K}(k_0)\tilde{D}_0(k_0)$ does not exist.

Now we follow replacing $\tilde{K}$ by the leading term in the perturbation expansion (now in terms of β), which is named, as said, ladder approximation. We remark, as a comment on the reliability of such a procedure, that in [7–9] a rigorous analysis shows that the spectral properties calculated in the ladder approximation are maintained.

Using $K=D_0^{-1}-D^{-1}$, we obtain

$$\tilde{K}(p,q,k)|_{O(\beta=0)}=\frac{\langle\varphi^4\rangle-3\langle\varphi^2\rangle^2}{2\langle\varphi^2\rangle^2[\langle\varphi^4\rangle-\langle\varphi^2\rangle^2]}=R, \quad (17)$$

i.e., constant (not depending on β), local in space and time. Writing $[1-A]^{-1}=1+A[1-A]^{-1}$ we get

$$(\tilde{f},\tilde{D}_0(k_0)\tilde{f})=\int \bar{\tilde{f}}(\vec{p})\tilde{D}_0(p,q,k_0)\tilde{f}(\vec{q})\ \ dpdq + \frac{R'}{1-R'I_D}\left(\int \bar{\tilde{f}}(\vec{p})\tilde{D}_0(p,q,k_0)dpdq\right)\times\left(\int \tilde{D}_0(p',q',k_0)\tilde{f}(\vec{q'})dp'dq\right), \quad (18)$$

where $dp\equiv d^{d+1}p$, etc., $R'=R/(2\pi)^{2(d+1)}$ and

$$I_D=\int dq'dq\tilde{D}_0(q',q,k_0)=2(2\pi)^{d+1}\int dp\tilde{S}_2(p)\tilde{S}_2(k_0-p_0,-\vec{p}).$$

Thus, the singularity, and so the bound state, comes for $R'I_D=1$. We follow the calculations separating in the expression for $\tilde{S}_2(p)$ the dominant one-particle contribution (as usual: we take the Lehmann spectral representation, see, e.g., [6,7])

$$\tilde{S}_2(p)=(2\pi)^d\tilde{c}_2(\vec{p})\frac{\sinh M(\vec{p})}{\cosh M(\vec{p})-\cos p_0}+\int_{3M-\varepsilon}^{\infty}\frac{\sinh E}{\cosh E-\cos p_0}d\eta(E,\vec{p})$$

where $\tilde{c}_2(\vec{p})=\partial\tilde{\Gamma}(p_0=i\xi,\vec{p})/\partial\xi|_{\xi=M(\vec{p})}$. We have, for the (first) dominant term, $\tilde{c}_2(\vec{p})=\langle\phi^2\rangle/(2\pi)^d+O(\beta)$. Thus, for the computation of (the leading part of) I_D we use $\tilde{S}_2(p)=\langle\phi^2\rangle\sinh M(\vec{p})/[\cosh M(\vec{p})-\cos p_0]$. Hence,

$$I_D=2(2\pi)^{d+1}\int dp\langle\phi^2\rangle^2\frac{(e^{M(\vec{p})}-e^{-M(\vec{p})})^2}{(e^{M(\vec{p})}+e^{-M(\vec{p})}-e^{ip_0}-e^{-ip_0})(e^{M(\vec{p})}+e^{-M(\vec{p})}-e^{ik_0-ip_0}-e^{-ik_0+ip_0})}.$$

Writing $ik_0=2M-\varepsilon$, we get (taking the leading term $\beta=0$) $I_D=2(2\pi)^{2(d+1)}\langle\phi^2\rangle^2/(1-e^{-\varepsilon})$. And for the eigenvalue equation $R'I_D=1$ we obtain

$$1-e^{-\varepsilon}=\frac{\langle\phi^4\rangle-3\langle\phi^2\rangle^2}{\langle\phi^4\rangle-\langle\phi^2\rangle^2}\equiv\zeta, \quad (19)$$

which leads, if $\zeta>0$, to the bound-state mass

$$M^*=2M+\ln(1-\zeta), \quad (20)$$

(we have the same bound-state mass expression of [9], there calculated for a ferromagnetic system with continuous spin, nearest-neighbor interaction only and even single spin distribution). ■

Now we analyze ζ. For simplicity we keep $\mathcal{P}(\varphi)=(a_6/6)\varphi^6+(a_4/4)\varphi^4$, but similar analysis easily follows for general (even) polynomials. Let us investigate the conditions on a_6 and a_4 which give us $\langle\varphi^4\rangle>3\langle\varphi^2\rangle^2$. Denoting φ by x or $y\in\mathbb{R}$, the correlation inequality may be written as

$$\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[-v(x)-v(y)](x^4+y^4-6x^2y^2)\,dx\,dy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[-v(x)-v(y)]\,dx\,dy}>0,$$

where $v(x)=\{\beta\lambda^2/8[a_6^2x^{10}+2a_6a_4x^8+a_4^2x^6]-\lambda/4[5a_6x^4+3a_4x^2]\}$. Using polar coordinates $x=r\cos\theta$, $y=r\sin\theta$, the inequality becomes (the numerator)

$$\int_0^{\infty}\int_0^{2\pi}e^{-v(r\cos\theta)-v(r\sin\theta)}(\cos^4\theta+\sin^4\theta-6\sin^2\cos^2\theta)r^5\,dr\,d\theta>0.$$

We estimate the contribution of the "small field" region (small r; the contribution of large r is subdominant) introducing the change of variables $e^{i\theta}=z$, and search for conditions which make positive the θ (i.e., z) integration, and so the whole expression. The z integration is over a unit circle on the complex plane, with center in zero, and may be calculated by the theory of residues. Considering small β and λ, the leading term after the z integration is $\lambda\pi 5a_6r^4/32$, and so positive if $a_6>0$. Namely, we shall have a bound state for a_4 positive or negative (in contrast with the results for the small noise regime).

IV. CONCLUDING REMARKS

We investigate some basic dynamical properties of the extensively used time-dependent GL model, analyzed when submitted to intense noise. The results obtained here determine the time relaxation rate of some correlation functions, and so, they are directly related to experimentally observable variables. We show a mass gap in the spectrum of the generator of the dynamics, and describe the one-particle and the two-particle bound-state masses, which shall increase with the noise strength. The contrast with previous results from the small noise regime [4,5] (there, the masses decay as we increase the noise intensity) establishes the existence of an interesting transient region (and poses the question of some noise-induced phase transition).

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